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# 73<sup>rd</sup> MORS Symposium / WG – 8 / 22 Jun 2005

## *Gains, losses, and thresholds of influence within a social network: A modeling approach*

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# Overview



- Problem Statement
- Social Sciences and Network Flows
  - Initial mappings
  - Gains, losses, and thresholds
  - Relationships to network flow formulations
- Notional Example
  - Generalized Network Flow Problem (GFP)
  - Post-optimality analyses



# Problem Statement



- Extend previous methodologies to generate and analyze courses of action applied to networks of individuals
- Overall goal - 'shaping intentions' through influence
  - ... in the context of military psychological operations that strive to influence an adversary's "... emotions, motives, reasoning, and ultimately, their behavior..." in order to achieve a given political goal. (JP 3-13, 1998:II-4)
- The means - extend previous SNA and OR mappings

# Current Mappings

<i>Social Closeness Terms</i>	<i>Flow Model Properties</i>
People or groups	Nodes (sinks, sources, or transshipment)
Connectivity or affinity	Capacitated arcs (or edges) between nodes
Social Closeness	Capacity
Influence	Commodity
Potential Influence	Magnitude of flow
People or groups initiating influence in the network	Source(s)
Target people or groups to be influenced	Sink(s)
People or groups involved in influencing	Transshipment node(s)
Multi-Criteria within a shared context	Multi-Commodity, contexts share capacity
Multi-Context or Multi-Criteria in different contexts	Multiple independent single-commodity models for each context or criteria



# Underlying Assumptions



- Renfro mappings are appropriate
- Accurate and complete network data
- Amount of influence generated by COA is measurable
  - Interpretation of influence amount is inviolate among individuals and their interactions
- Directed network mimics the anticipated operational channels of communication
  - No discussion or interaction, as seen in traditional SNA approaches, is modeled



# Research Focus



- “Gains and losses represent predispositions, communication problems, and other similar factors based on the specific scenario under consideration.” (Renfro, 2001:67)
- “Thresholds can also be set for cases where individuals or groups require a minimum level of influence before they take a specific course of action.” (Renfro, 2001:67)
- Requires Generalized Network Flow
  - Arcs may consume or generate flow
  - Seen in power networks, canals, transportation of perishable commodities, and cash management (Ahuja, *et al*, 1993:8)
  - Develop maximum flow and minimum cost, maximum flow approaches





# Influence

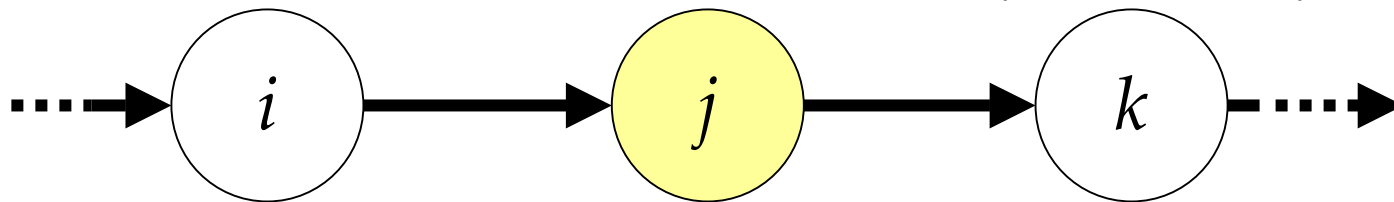


- As a commodity...
  - Transfers between individuals to enable group opinion formation (Friedkin and Cook, 1990)
  - Contagion of behavior (Leenders, 2002)
  - Diffusion of innovations (Valente, 1996)
  - Propagation of extremist opinions (Amblard and Deffuant, 2004)
  - Basis for interpersonal power of one individual over another (French, 1956)

# Network Flow

“Outflow minus inflow must equal supply (or demand)”

Amount of flow from node  $i$  to node  $j$  on arc  $(i, j)$  is  $x_{ij}$



Mass Balance Constraints (Three cases)

Supply node: outflow  $>$  inflow  $\Rightarrow$  outflow = inflow +  $b_j$

$$x_{jk} - x_{ij} = b_j$$

Thresholds

Demand node: outflow  $<$  inflow  $\Rightarrow$  outflow = inflow -  $b_j$

$$x_{jk} - x_{ij} = -b_j$$

Transshipment node: outflow = inflow

$$x_{jk} - x_{ij} = 0$$

(Ahuja, Magnanti, and Orlin, 1993:5)



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# Network Flow

## *Target Flow*



- Given the following...
  - A network structure
  - Social closeness measures for all arcs  $(i, j)$
- The objectives...
  - Identify key actors that serve as ultimate targets of influence
  - Identify actors that are accessible and likely to propagate influence through the network
  - Identify the minimum amount of influence required

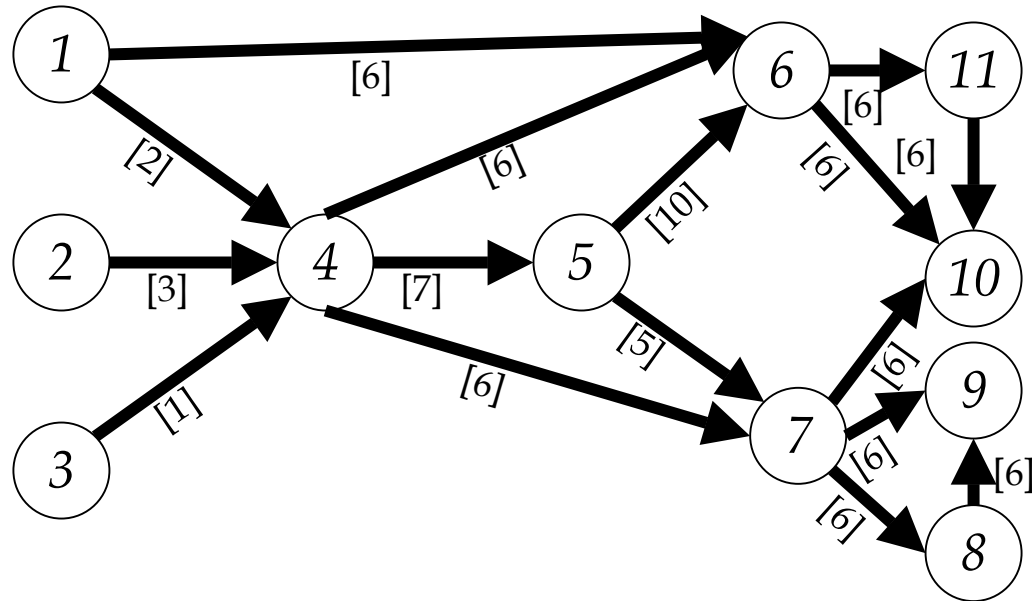


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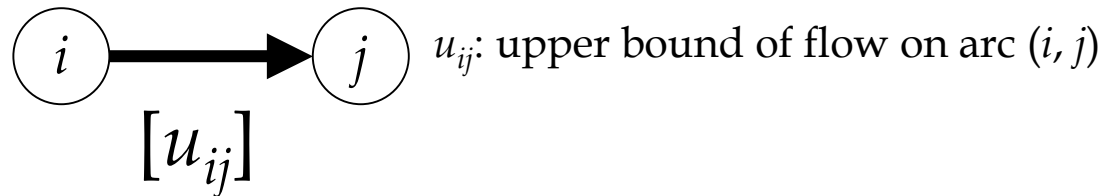


# Notional Network

## Target Flow



Legend



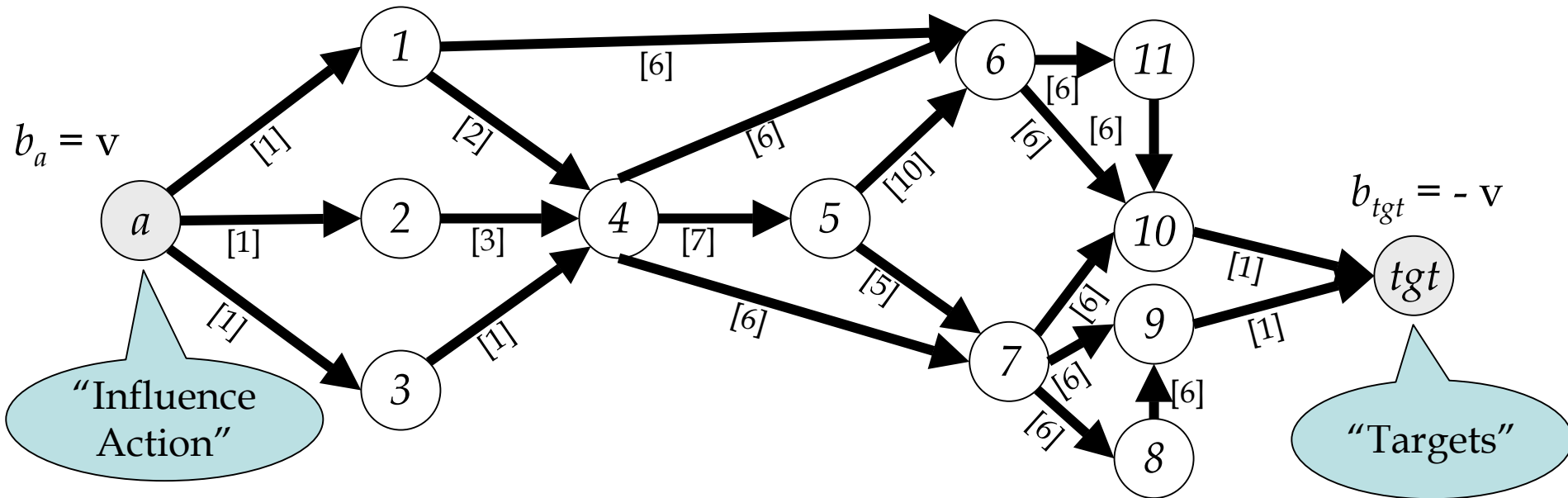


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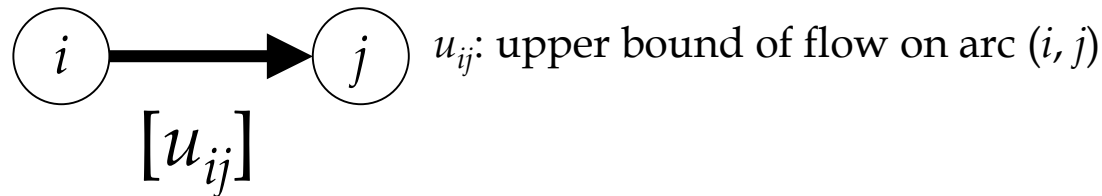


# Notional Network

## Target Flow



Legend



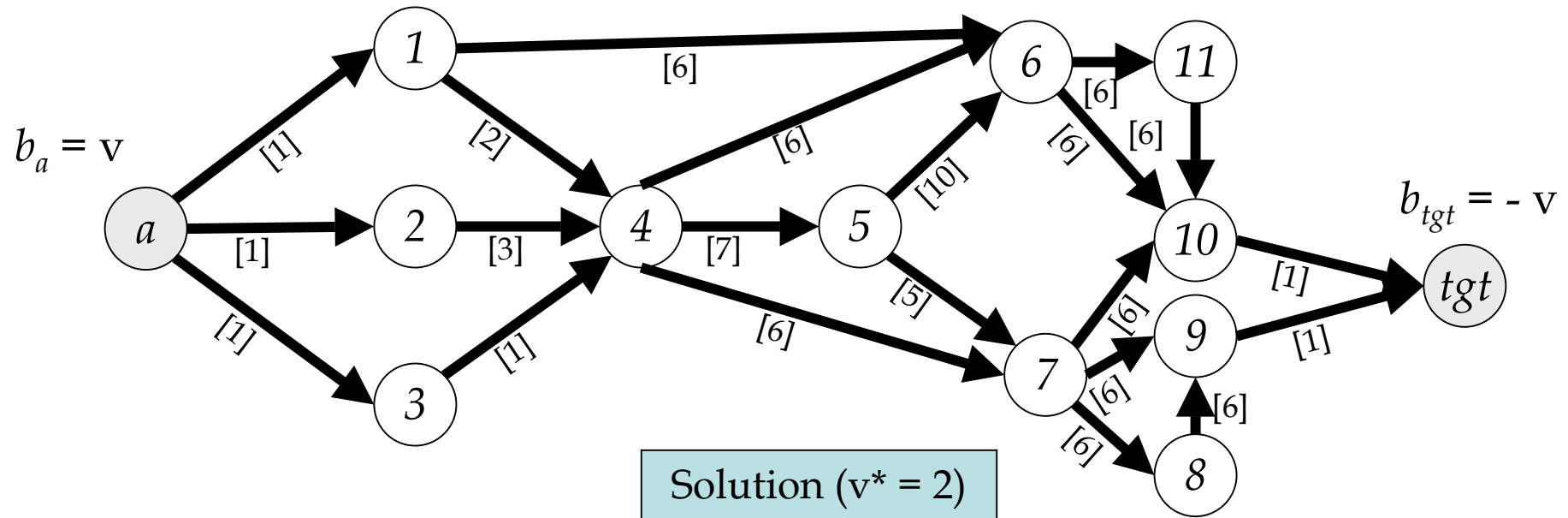


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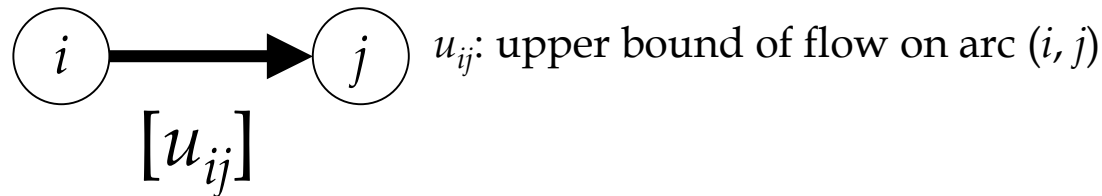


# Notional Network

## Target Flow



Legend





# Gains



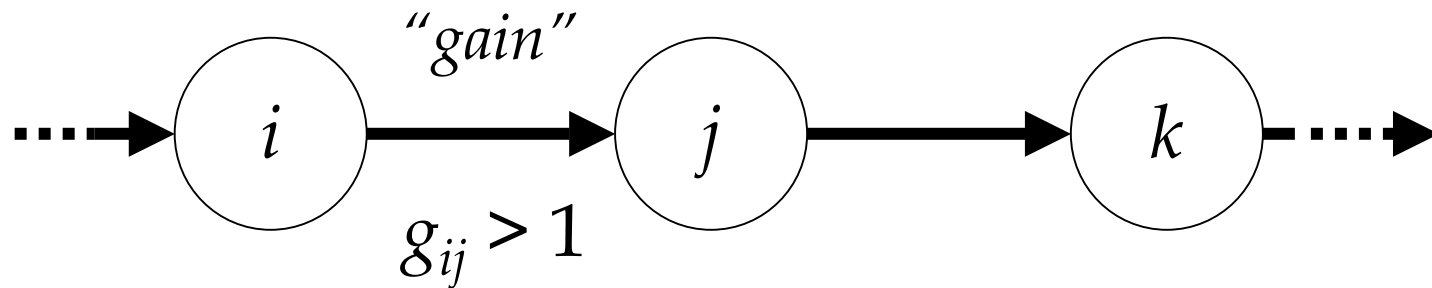
- Influence is not necessarily equitable between two actors (Renfro, 2001:103)
- Predispositions of individuals favoring influence (Renfro, 2001, 88)
- "... person's opinions may be tugged in various directions by the influences of their significant others and that individuals deal with these cross-pressures by shifting their opinions into positions where pressures are balanced." (Friedkin and Cook, 1990:130)
- Interpersonal power – "maximum force which A can induce on B minus the maximum resisting force which B can mobilize in the opposite direction" (French, 1956:183-4)
  - Five Bases: Attraction, Expert, Reward, Coercive, Legitimate
  - Must be measured from A's and/or B's perspective



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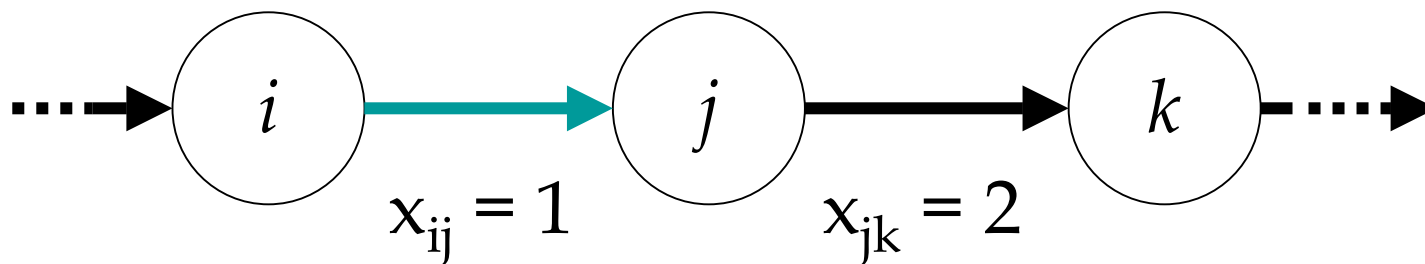


# Gains



For node  $j$ , "out - in" is represented by  $x_{jk} - g_{ij}x_{ij} = 0$

Given  $x_{ij} = 1$  and  $g_{ij} = 2 \Rightarrow x_{jk} = 2$







# Losses

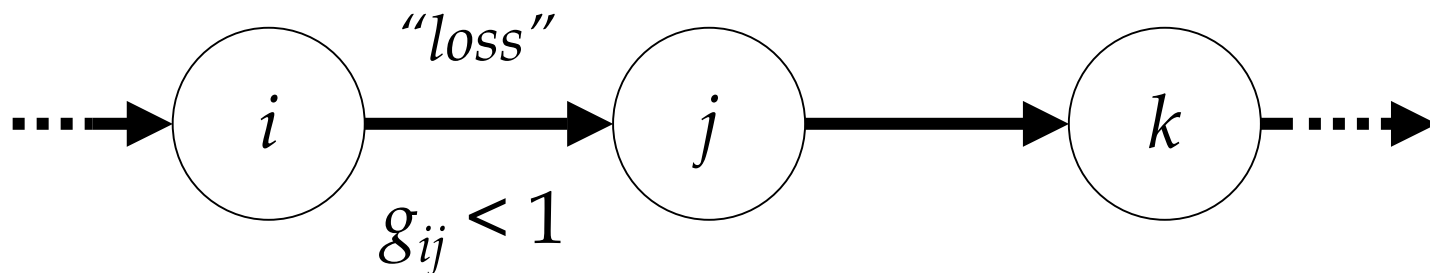
- Theory presented by (French, 1956) also applies
- “... communication problems such as misunderstanding the message.” (Renfro, 2001:88)
- (Lopez, *et al*, 2002) link organizational structure to efficiency of information flow
- (Friedkin and Johnsen, 2002) analyze impact of organizational structure and span of control
  - Mitigation via “Fayol’s gangplanks”
  - Traces back to book by Williamson (1971)



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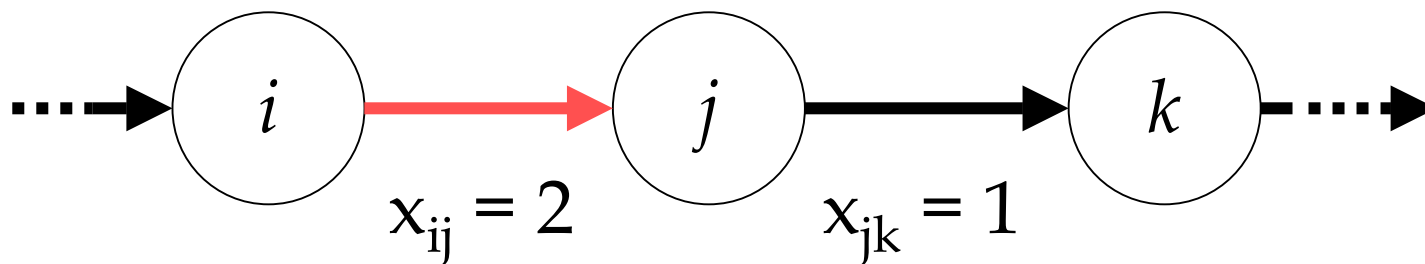


# Losses



For node  $j$ , "out - in" is represented by  $x_{jk} - g_{ij}x_{ij} = 0$

Given  $x_{ij} = 2$  and  $g_{ij} = 0.5 \Rightarrow x_{jk} = 1$





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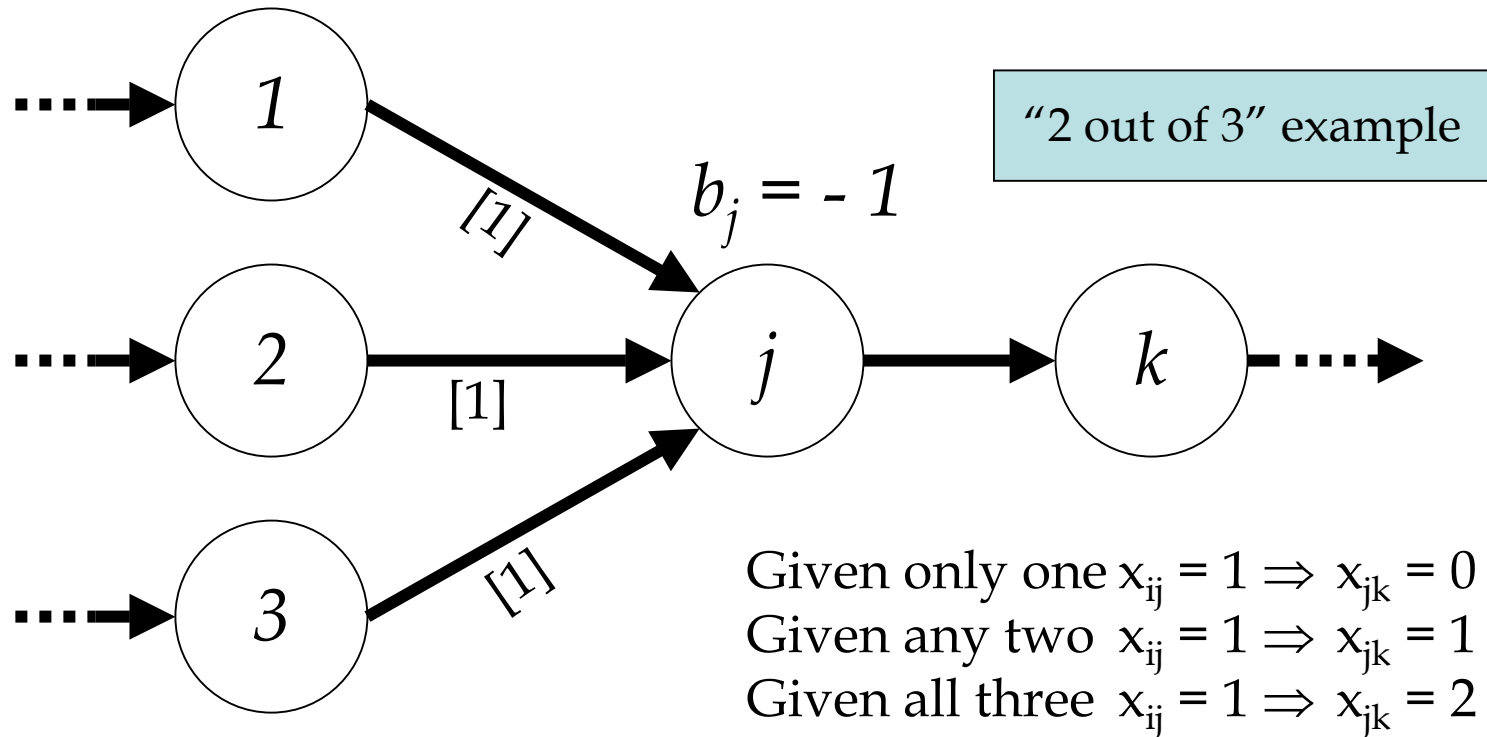
# Thresholds

- “Models of collective behavior are developed for situations where actors have two alternative and the costs and/or benefits of each depend on how many other actors choose which alternative.” (Granovetter, 1978:1420)
  - Threshold – number or proportion required at point where benefits exceed costs for that actor
  - Innovations, rumors and diseases, strikes, voting, educational attainment, leaving social occasions, migration, and experimental social psychology (1423-4)
- (Valente, 1996) developed a (social) network threshold model for diffusion of innovations
- Two modeling options are presented



# Thresholds

*“m out of n”*



Assuming one unit of flow from any  $i$  to  $j$ , at least two of the three individuals must “influence”  $j$  before  $j$  will “influence”  $k$

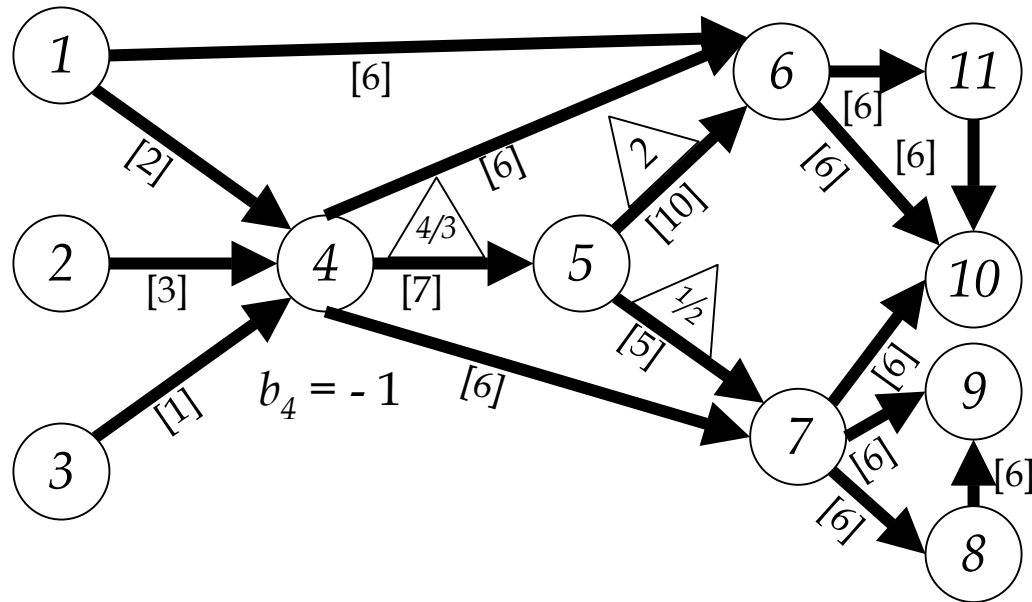


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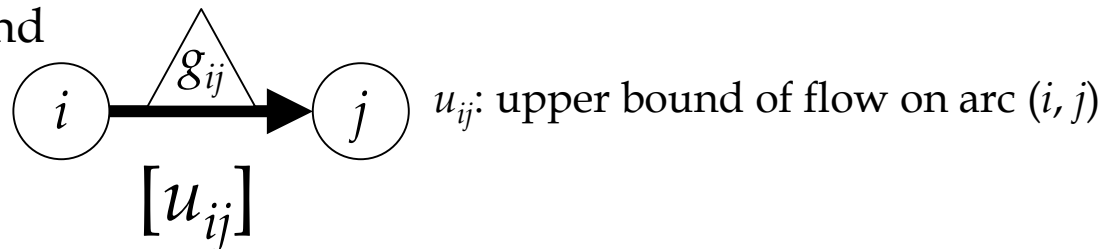


# Notional Network

## Maximum Flow



Legend



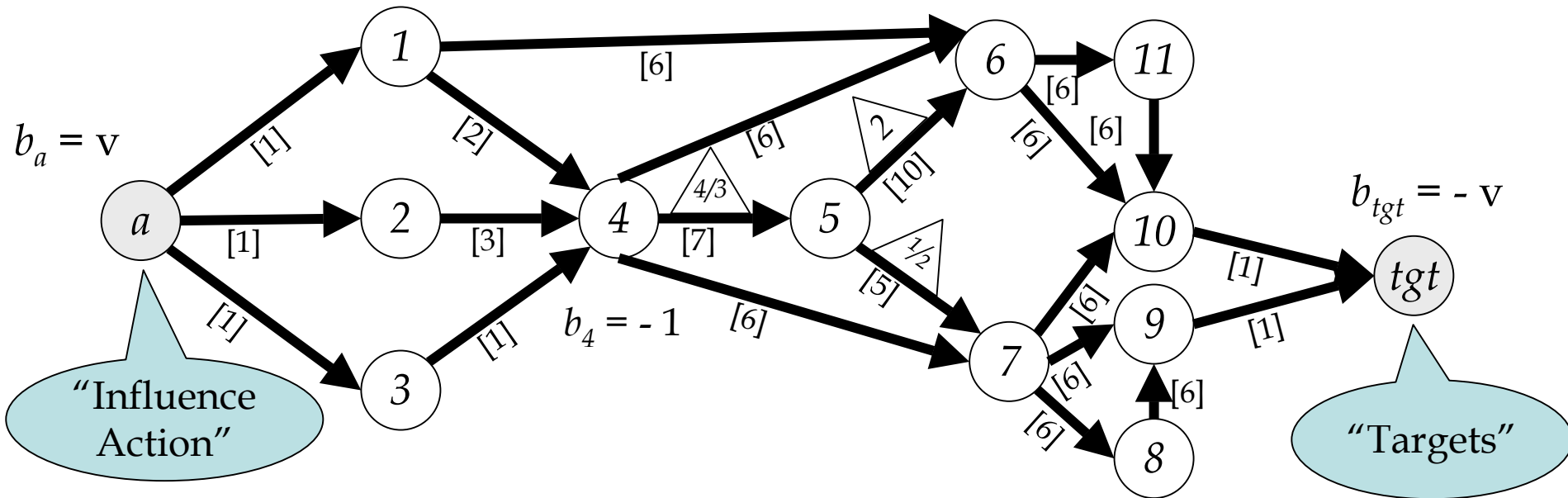


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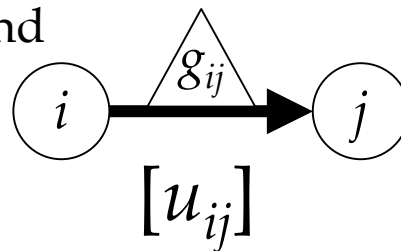


# Notional Network

## Maximum Flow



Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$

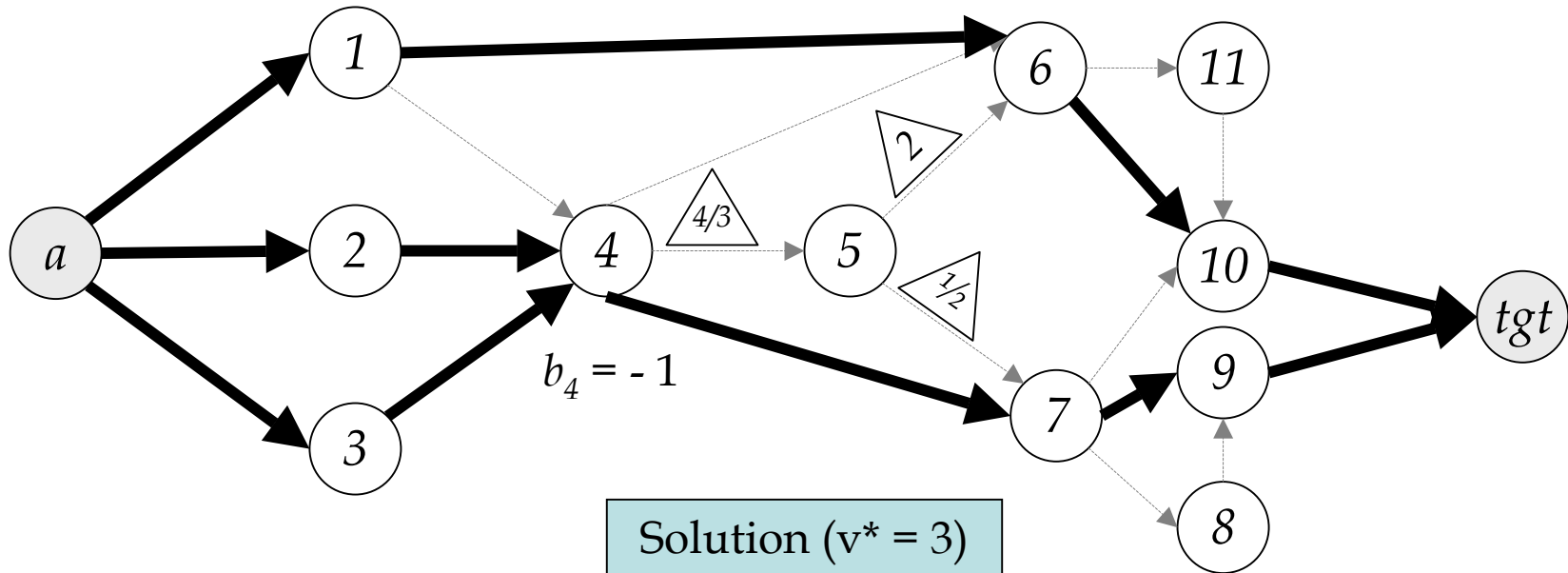


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# Notional Network

## Maximum Flow





# Minimum cost, maximum flow



- External Costs – Course of Action
  - Represent risk friendly forces are subjected to when implementing the COA
    - Node “*a*” to all initial target nodes - execution
    - Target nodes to “*tgt*” node - observation
- Internal Costs
  - Represent risks perceived by individuals within the network
  - Operational – Fear of compromise
  - Personal – Fear of retribution
  - May also apply to individuals external to the network of interest



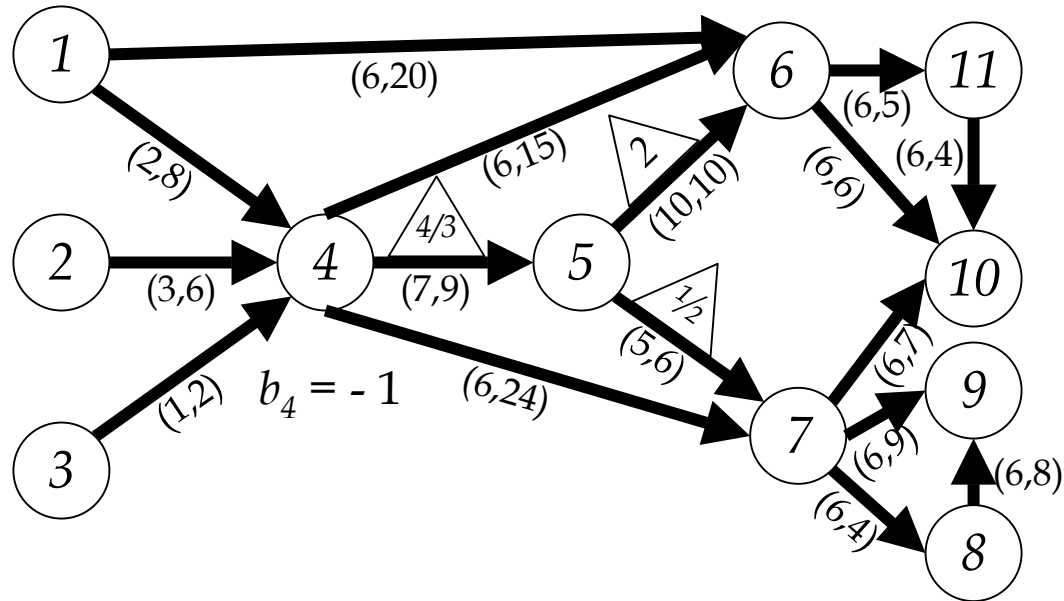


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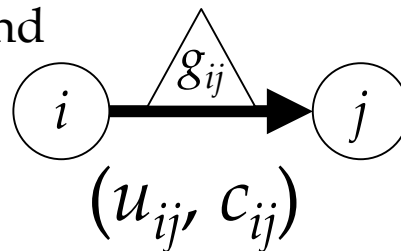


# Notional Network

## *Minimum cost, maximum flow*



Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$   
 $c_{ij}$ : cost per unit flow on arc  $(i, j)$   
 $g_{ij}$ : gain/loss factor for arc  $(i, j)$

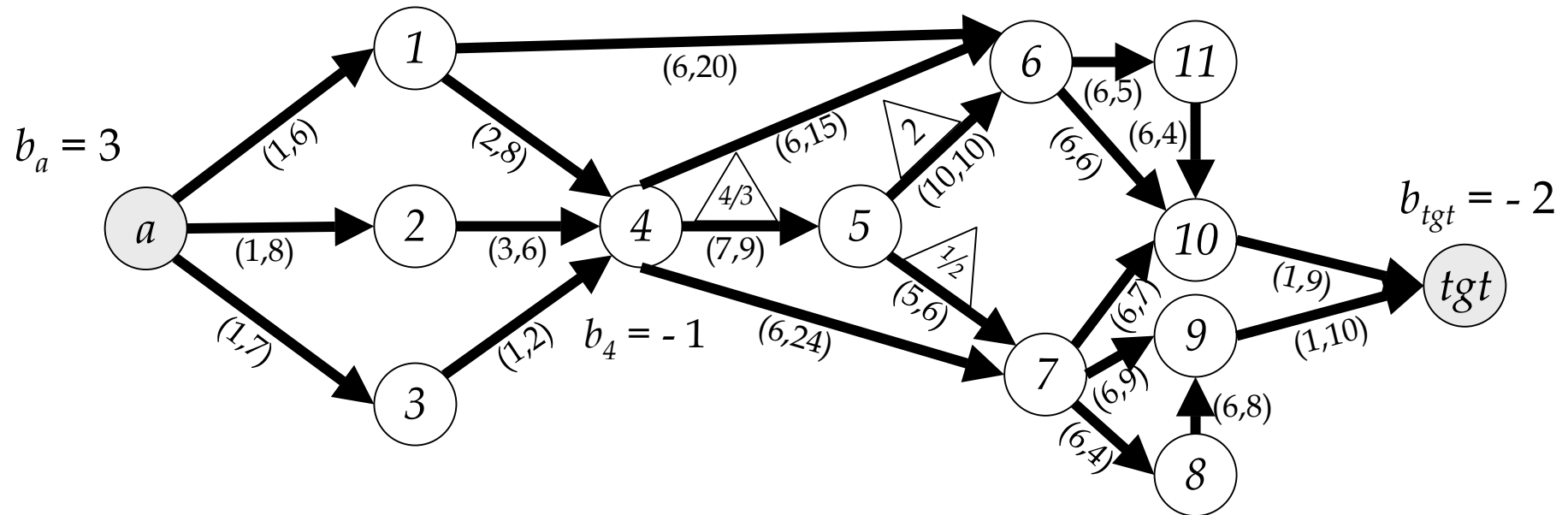


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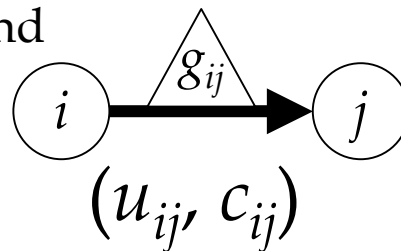


# Notional Network

## *Minimum cost, maximum flow*



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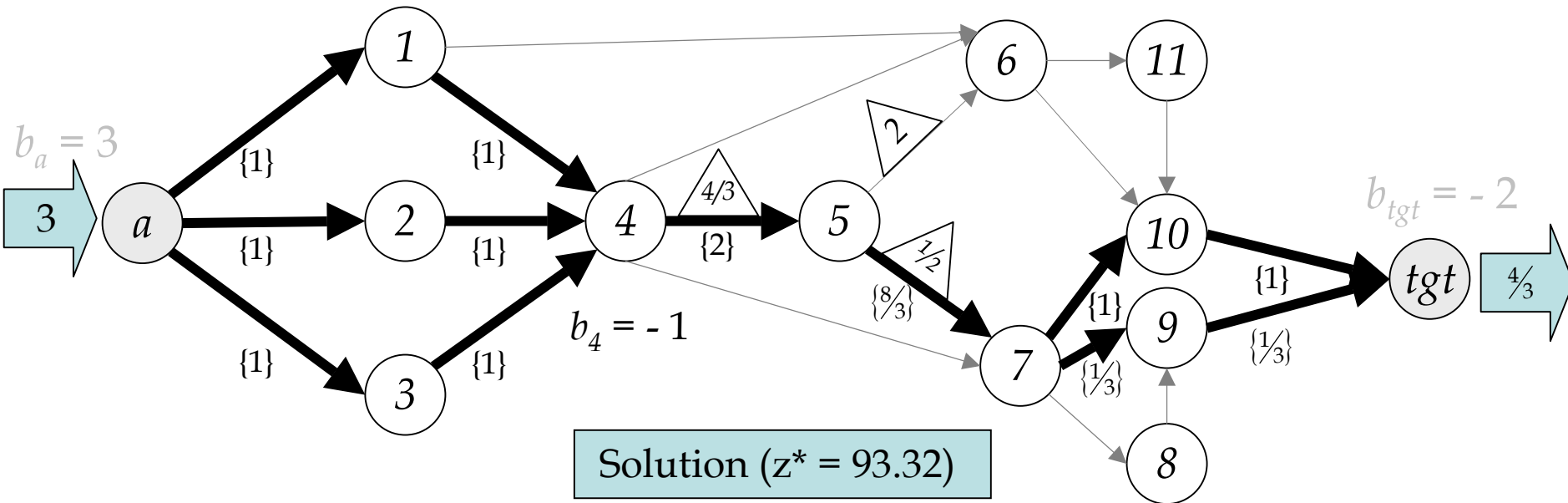


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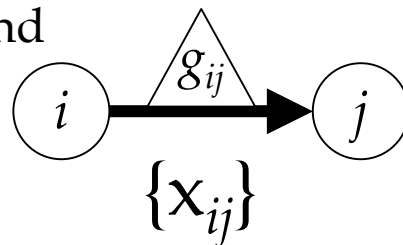


# Notional Network

## *Minimum cost, maximum flow*

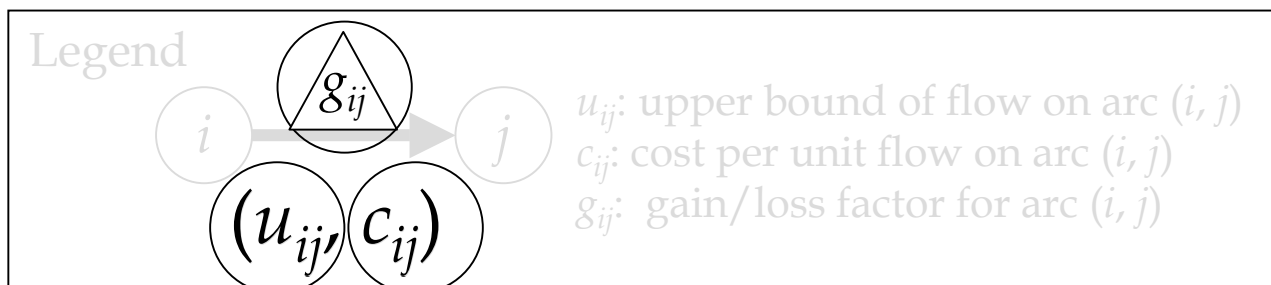
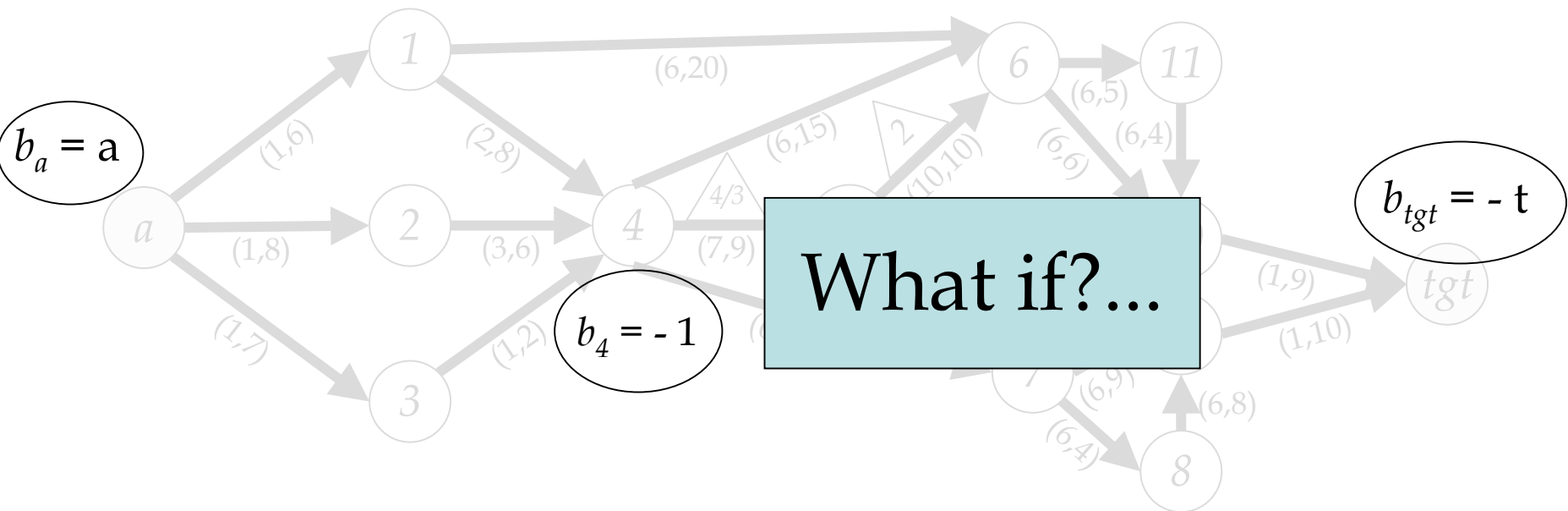


Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$   
 $x_{ij}$ : amount of flow on arc  $(i, j)$

# Post-optimality Analysis





# Post-optimality Analysis



- Objective Function Coefficients
- Right-hand side
  - Thresholds,  $b_a$ ,  $b_{tgt}$
  - Upper bounds (if included as a constraint)
- Technological coefficients
  - Gains and losses
- Parametric and multiple changes



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# Post-optimality Analysis

## *Objective Function Coefficients*

Currently in basis

Potentially of interest

Variable	Current	Allowable Increase	Allowable Decrease
X1_4	8	0	3
X1_6	20	3	0.339
X2_4	6	$\infty$	0
X3_4	2	5	31.662
X4_5	9	0.339	12.662
X4_6	15	$\infty$	3
X4_7	24	$\infty$	13.339
X5_6	10	$\infty$	29.823
X5_7	6	0.254	9.498
X6_10	6	$\infty$	0.339
X6_11	5	12.662	3.339
X7_8	4	15	3
X7_9	9	3	1.015
X7_10	7	0.339	$\infty$
X8_9	8	$\infty$	3
X9_tgt	10	$\infty$	1.015
X10_tgt	9	3	$\infty$
X11_10	4	$\infty$	3.339
Xa_1	6	0	$\infty$
Xa_2	8	$\infty$	0
Xa_3	7	5	$\infty$

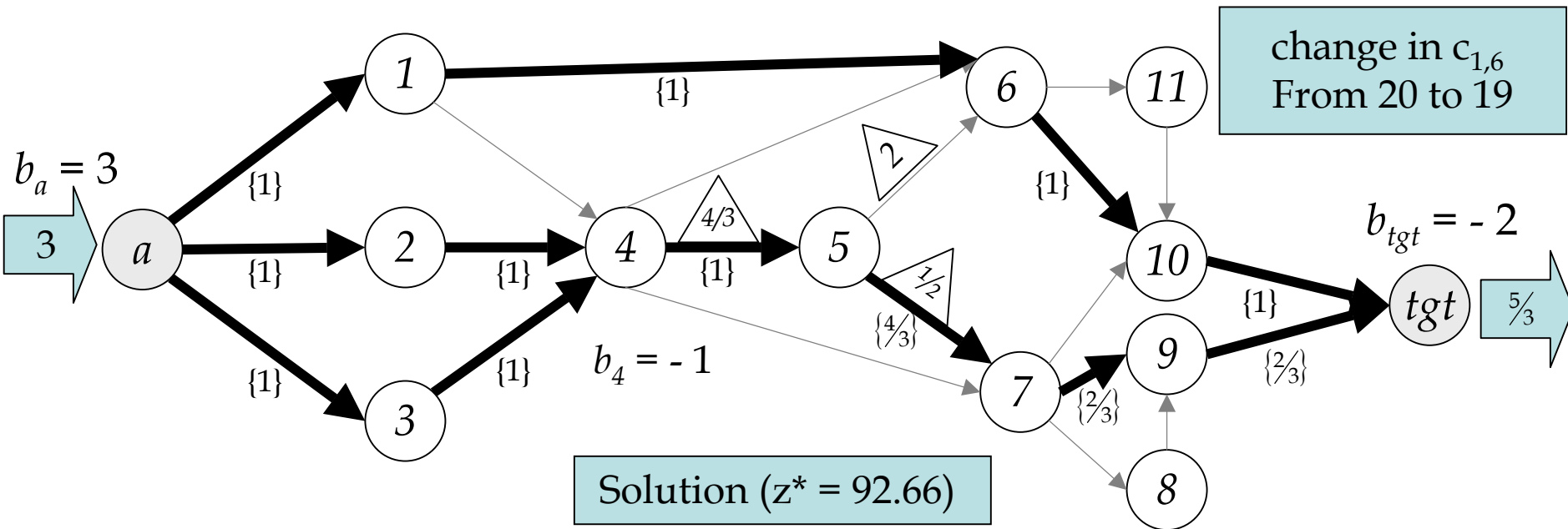


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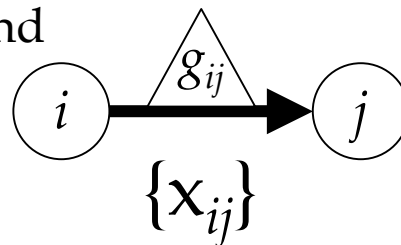


# Notional Network

## Cost Coefficients



Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$   
 $x_{ij}$ : amount of flow on arc  $(i, j)$



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# Post-optimality Analysis

## *Right-hand Side*



Row (Node)	Current Value	Allowable Increase	Allowable Decrease
a	3	0	0.499
1	0	1	0.499
2	0	1	0.499
3	0	0	0
4	-1	1	0.499
5	0	1.33	0.67
6	0	0.67	0
7	0	0.67	0.33
8	0	0	0.33
9	0	0.67	0.33
10	0	0.67	0.33
11	0	0	0
tgt	-2	0.67	$\infty$

Potentially of interest



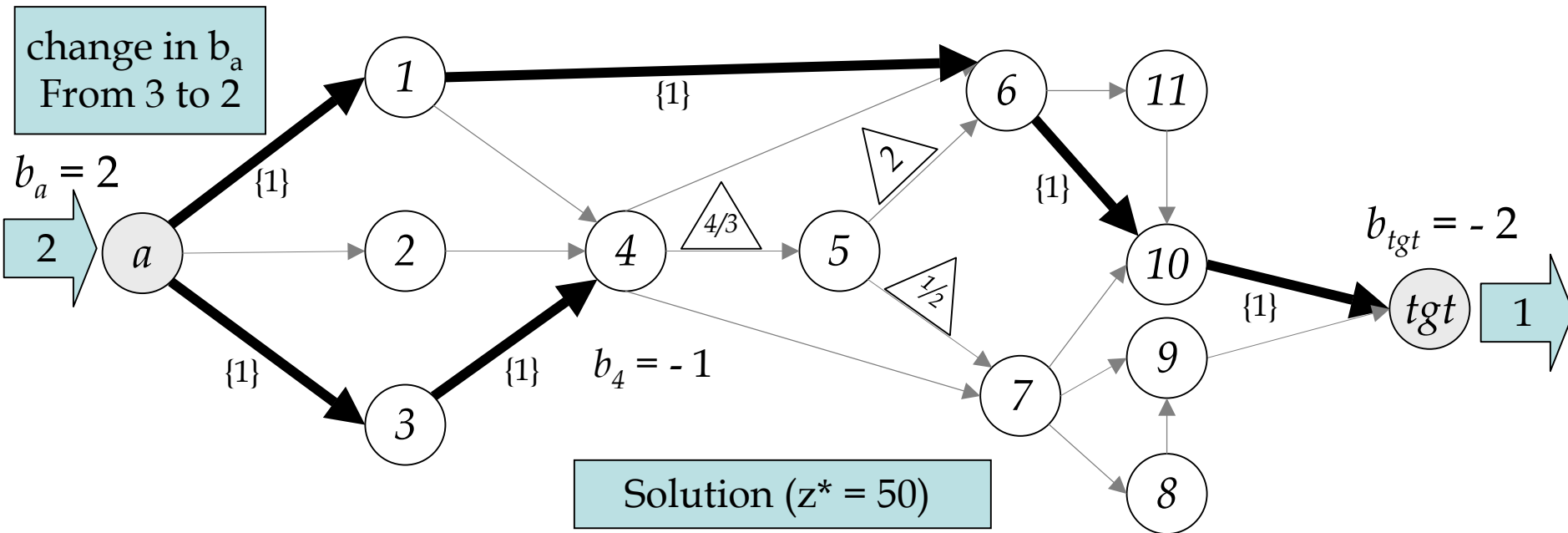


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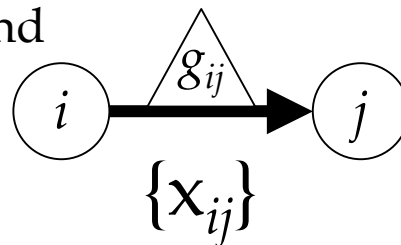


# Notional Network

## *Right-hand Side*



Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$   
 $x_{ij}$ : amount of flow on arc  $(i, j)$

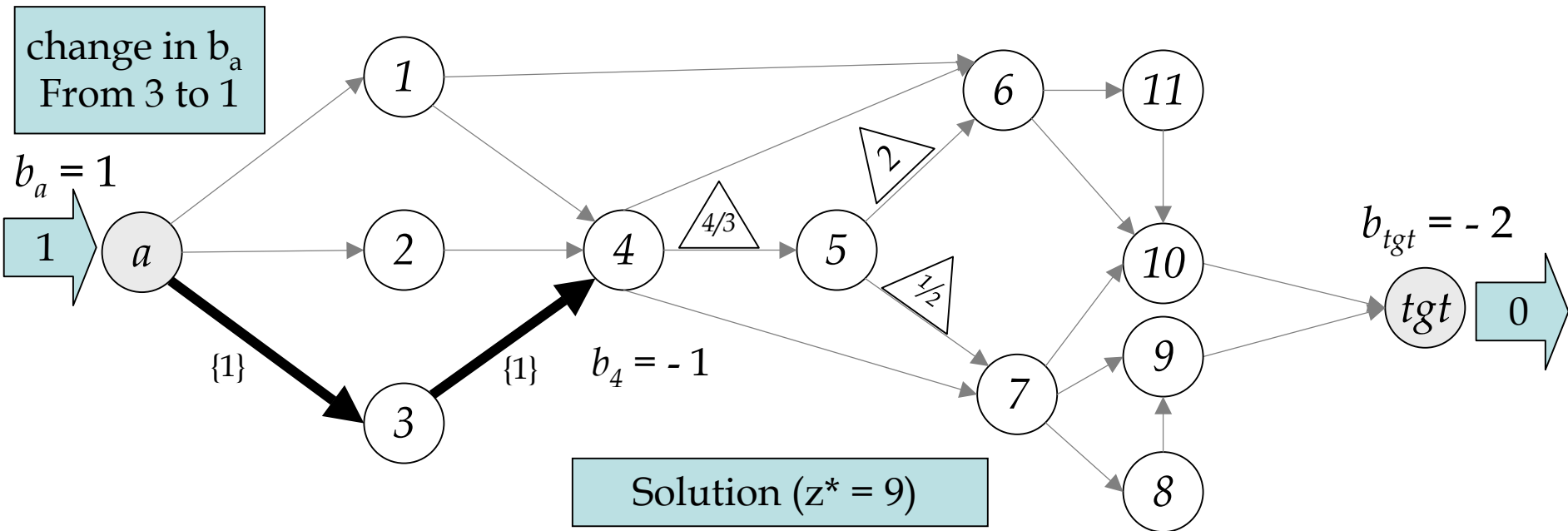


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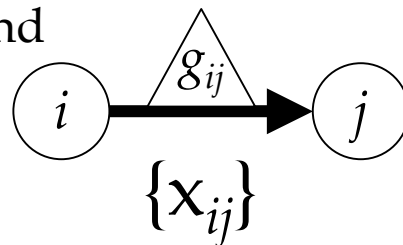


# Notional Network

## *Right-hand Side*



Legend



$u_{ij}$ : upper bound of flow on arc  $(i, j)$   
 $x_{ij}$ : amount of flow on arc  $(i, j)$



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# Post-optimality Analysis

## *Arc Capacities*

<i>Arcs with non-zero flow</i>		<i>Arcs with zero flow</i>	
Variable	Current (Range)*	Variable	Current (Range)
X1_4	2 (1, $\infty$ )	X1_6	6 (6, $\infty$ )
X2_4	3 (2, $\infty$ )	X4_6	6 (6, $\infty$ )
X3_4	1 (0, $\infty$ )	X4_7	6 (6, $\infty$ )
X4_5	7 (5, $\infty$ )	X6_11	6 (6, $\infty$ )
X5_7	5 (2 $\frac{1}{3}$ , $\infty$ )	X7_8	6 (6, $\infty$ )
X7_9	6 (5 $\frac{2}{3}$ , $\infty$ )	X8_9	6 (6, $\infty$ )
X7_10	6 (5, $\infty$ )	X11_10	6 (6, $\infty$ )
X9_tgt	1 (2 $\frac{2}{3}$ , $\infty$ )	X5_6	10 (10, $\infty$ )
X10_tgt	1 (2 $\frac{2}{3}$ , 1 $\frac{1}{3}$ )	X6_10	6 (6, $\infty$ )
Xa_1	1 (0, $\infty$ )		
Xa_2	1 (0, 1)		
Xa_3	1 (0, 0)		

\* The (Range) indicates allowable decrease,  $d$ , and allowable increase,  $i$ , denoted by  $(d, i)$ .

# Post-optimality Analysis

## *Technological Coefficients*

Constraint $i$	Variable $j$	Acceptable Change	Conditions
Binding	Basic	$0 \leq \Delta a_{i,j} \leq 0$	--
	Non-Basic	$\frac{c_j - \mathbf{c}_B \mathbf{B}^{-1} \mathbf{a}_j}{w_i} \leq \Delta a_{i,j} \leq \infty$	For all $\leq$ constraints
		$-\infty \leq \Delta a_{i,j} \leq \frac{c_j - \mathbf{c}_B \mathbf{B}^{-1} \mathbf{a}_j}{w_i}$	For all $\geq$ constraints
Non-binding	Non-Basic	$-\infty \leq \Delta a_{i,j} \leq \frac{x_{n+i}}{x_j}$	For $\leq$ constraint ( $x_{n+i}$ = slack)
		$\frac{-x_{n+i}}{x_j} \leq \Delta a_{i,j} \leq \infty$	For $\geq$ constraint ( $x_{n+i}$ = surplus)

Although many optimization software packages do not provide this capability, similar analyses on the technological coefficients (i.e. the gains and losses) may be performed

(Hartley, 1976:163-4; Bazaraa, et al, 1990:281-3)



# Conclusions



- Social Sciences and Network Flows
  - Initial mappings
  - Gains, losses, and thresholds
  - Relationships to network flow formulations
- GFP and Notional Examples
  - Advantages - Post-optimality analyses
  - Disadvantages – Data, Deterministic, ...
- Attractive option to analyze, better understand, and predict behavior of non-cooperative networks in response to external influence



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# Backups/Old Slides



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# 712-B



		<b>GOVERNMENT DISCLOSURE FORM</b> <b>73<sup>rd</sup> MORS Symposium</b> US Military Academy, West Point, NY – 21-23 June 2005		<b>712-B</b>	
<b>DEADLINE: 9 MAY 05 – Fax to 703-933-9066</b>			<b>MORS P #:</b> <small>For office use only 117384</small>		
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Principal Author's Organization and address: <b>AFIT/ENS, Building 641 2950 Hobson Way Wright Patterson AFB OH 45433-7765</b>			Phone: <b>937-305-1662</b> Fax: <b>937-656-1913</b> Email: <b>j.hamill@afit.edu</b>		
Title of Presentation: <b>Gains, losses, and thresholds of influence within a social network: A modeling approach</b>					
Presentation will be presented in:	WG 21 - 33 or WG name:	<b>WG-8</b>	Special Session: 1, 2 or 3	Other: Poster, Demo, CD, Tutorial	
This presentation is being submitted in the form of: <input type="checkbox"/> SECRET <input type="checkbox"/> CONFIDENTIAL <input checked="" type="checkbox"/> UNCLASSIFIED					
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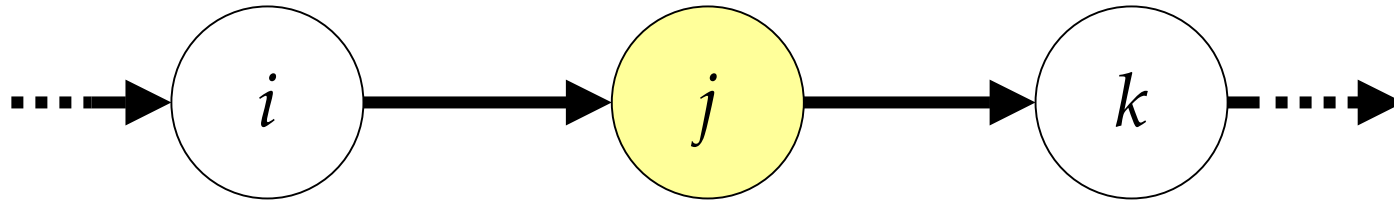


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# Network Flow

Flow bound constraints are the upper and lower limits of  $x_{ij}$



*Social Closeness* ( $S_{ij}$ ), measured by a value-focused thinking model, is defined as “the maximum potential influence one person or group ( $i$ ) has upon another person or group ( $j$ )...” in a given social network and under a given scenario. (Renfro, 2001:89)



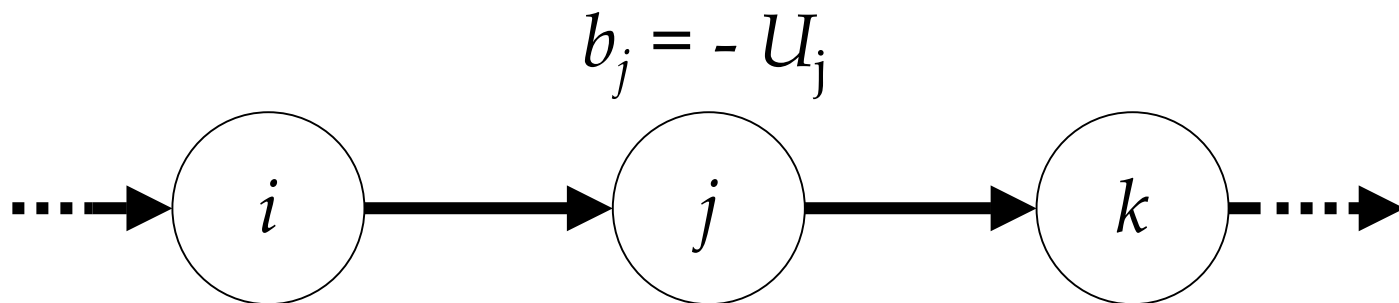


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# Thresholds

*“Absorbing node”*



$$U_j \geq \sum_{\{j:(i,j) \in A\}} s_{ij} \quad \text{Influence will *not* pass}$$

However, varying this input can have some interesting properties...